THE ASTROPHYSICAL JOURNAL, 276:1–3, 1984 January 1

TESTING THE STRONG EQUIVALENCE PRINCIPLE BY RADIO RANGING

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Received 1983 March 23; accepted 1983 June 10

ABSTRACT

Planetary range data offer the most promising means to test the validity of the Strong Equivalence Principle (SEP). Here we derive and compare analytical expressions for the perturbation in the "range" expected from an SEP violation predicted by the "variation-of-G" method and by the "two-times" approach. The dominant term in both expressions is quadratic in time. Analysis of existing range data should allow a determination of the coefficient of this term with a one-standard-deviation uncertainty of about 1 part in 10^{11} yr^{-1}.

Subject headings: cosmology — planets: general — relativity — radar astronomy

I. INTRODUCTION

The Strong Equivalence Principle, or SEP (Will 1979; Thorne, Lee, and Lightman 1973; Canuto and Goldman 1982*a*, *b*), demands that local physics, described in a local Lorentzian frame, be the same at any time and anywhere in the universe. A violation of the SEP could thus be claimed if one could prove that cosmological effects influenced the outcome of local experiments. Although one may consider a variety of experiments of a nongravitational nature, there is one gravitational experiment that appears to offer useful accuracy for testing a violation of the SEP of cosmological origin: the measurement, with atomic clocks, of the round-trip travel time ("two-way range") of radio signals transmitted from the Earth and reflected by other planets or transponded by spacecraft in heliocentric orbit. If the atomic clock measurements indicated that there were secular changes in the orbital periods of the Earth and the planet or spacecraft, not otherwise accounted for by "classical" effects on these orbits, then one would be led to infer that an SEP violation has been found.

The most suitable data presently available for such a test consist of the radar ranging measurements for the inner planets and, especially, the radio ranging measurements for the *Viking* landers on Mars. The latter ranging measurements have an intrinsic accuracy sufficient to determine the secular rate of change in orbital period, normalized to that period ("$\dot{P}/P$"), with a standard error of about 1 part in 10^{11} yr^{-1}, or less. However, a present lack of understanding of the systematic errors affecting the analysis of these data is preventing the achievement of this low standard error (Shapiro and Reasenberg 1980; Hellings 1982). It is important to gain this understanding since a violation of the SEP of cosmological origin would be expected to lead to inferred values of $\dot{P}/P$ of 5–10×10^{-11} yr^{-1}, the approximate span of values currently estimated for the Hubble constant whose inverse is a measure of the age of the universe. At present, the experimental bound on "nonclassical" contributions to $\dot{P}/P$ is of order 10^{-10} yr^{-1} (Reasenberg and Shapiro 1976).

The analysis of these ranging data requires that one employ a set of dynamical equations to describe the motion of bodies in the solar system. To detect most readily an SEP violation, it is useful to modify the standard SEP-conserving dynamical equations to include a parameterization of the effect for which one is searching. This parameterization is by no means unique: different theoretical models may involve different concepts of an SEP violation, resulting in different dynamical equations. As an example, we analyze more closely the interpretation of ranging to a planet in the hypothetical case in which only test-mass planets orbit the (point-mass) Sun. Such a planet, revolving about the Sun, can be considered as a clock governed solely by gravity: a gravitational clock. In the process of monitoring its period with an atomic clock, one measures directly the ratio of the periods of the two clocks. From this point of view, ranging experiments imply *two* clocks, based on two independent dynamics: gravity and electromagnetism (see, for example, Canuto and Goldman 1982*a*, *b*).

Two methods of parameterization have been proposed to seek evidence of an SEP violation (i.e., any nonlinear relation between the "natural" units of the two types of clocks); one is the "variation-of-G" method and the other is the "two-times" method. The theoretical aspects entailed by a consistent implementation of the two methods, specifically the categorization of gravity theories into two types of metric theories, as well as the implications for nongravitational physics have recently been discussed by Adams *et al.* (1983) and by Canuto and Goldman (1983). Here we discuss the two methods and show that for the data analyzed thus far, they lead to equivalent results. However, differentiation between the two would be important if a violation were to be found, because it then would restrict SEP-violating theories and could allow useful inferences to be made about the physical meaning and implications of an SEP violation. The two methods to be discussed do in fact offer very different views of an SEP violation.

II. METHOD I

The basic assumption of Method I (the "variation-of-G" method) is that an SEP violation does not change the form of Newton's equations in atomic units, except that the gravitational coupling becomes a time-dependent function, so that

$$\frac{d^2x^k}{dt^2} = -\frac{G(t)M_0}{r^3}x^k , \quad (1)$$

where $G(t)$ denotes the (assumed) time-dependent gravitational constant and where M_0 denotes the (constant) mass of the Sun and x^k ($k = 1, 2, 3$) the Cartesian coordinates of the distance r between a (test-mass) planet and the Sun (see Shapiro 1964 and Shapiro *et al.* 1971; for motivation, see Dirac 1938). If we consider only first-order effects, more than adequate for the time intervals of current interest, we may substitute

$$G(t) = G_0 + \dot{G}_0 t \quad (2)$$

in equation (1) and obtain the first-order solution:

$$x^k(t) = \left(1 - \frac{\dot{G}_0}{G_0}t\right)x_0{}^k(t^*) , \quad (3)$$

where the variable t^* is defined by

$$t^* = t + \left(\frac{\dot{G}_0}{G_0}\right)t^2 , \quad (4)$$

where a dot denotes differentiation with respect to t, and where $x_0(t)$ is the solution to equation (1) corresponding to $G \equiv G_0$, a constant. The measured quantity τ, the round-trip time of a radio signal, can be represented adequately for our purposes by

$$c\tau = |\boldsymbol{x}_1 - \boldsymbol{x}_2| , \quad (5)$$

where c is the speed of light and $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ denote the vector positions of the Earth and the "target" planet and here are solutions of equation (1) in the limit that the planet masses are vanishingly small, an adequate approximation for present purposes. Since the ranging experiments are performed using atomic clocks, we identify τ with the time interval measured by an atomic clock, i.e., we assume equation (1) to be valid in *atomic units*. Substituting equations (3) and (4) into equation (5), we derive

$$\delta\tau \equiv \tau_{\rm I}(t) - \tau_0(t) = -t\tau_0\left(\frac{\dot{G}_0}{G_0}\right)\left[1 - t\left(\frac{\dot{\tau}_0}{\tau_0}\right)\right] \quad (6)$$

which represents the perturbation in the range caused by an SEP violation since τ_0 represents the round-trip time in the absence of an SEP violation.

III. METHOD II

In Method II Newton's equations are assumed to hold in their usual form in gravitational units (subscript E, for Einstein)

$$\frac{d^2x_{\rm E}{}^k}{dt_{\rm E}{}^2} = -\frac{G_0 M_0}{r_{\rm E}{}^3}x_{\rm E}{}^k , \quad (7)$$

where G_0 and M_0 are constant.

Since distances are measured by travel times of light signals, we assume the transformation between atomic and gravitational units is the same for both length and time intervals (see, also, Dirac 1974 and 1979):

$$dx_{\rm E}(t_{\rm E}) = \beta_a(t)dx(t) , \quad dt_{\rm E} = \beta_a(t)dt . \quad (8)$$

As before, we can write to first order

$$\beta_a(t) = 1 + \left(\frac{\dot{\beta}_a}{\beta_a}\right)_0 t , \quad \beta_a(t_0) = 1 , \quad (9)$$

where we introduce the function $\beta_a(t)$ to conform with previous usage (Canuto and Goldman 1982*a*, *b*). The variation $\delta\tau$ in the range given by equation (5) can then easily be evaluated as

$$\delta\tau \equiv \tau_{\rm II}(t) - \tau_0(t) = -t\tau_0\left(\frac{\dot{\beta}_a}{\beta_a}\right)_0\left[1 - \frac{1}{2}t\left(\frac{\dot{\tau}_0}{\tau_0}\right)\right] . \quad (10)$$

Using equations (8) and (9), we can also transform equation (7) into atomic units:

$$\frac{d^2x^k}{dt^2} = -\frac{G_0 M_0}{r^3}x^k + \left(\frac{\dot{\beta}_a}{\beta_a}\right)_0 \frac{G_0 M_0}{r^3}x^k t - \left(\frac{\dot{\beta}_a}{\beta_a}\right)_0 \frac{dx^k}{dt} . \quad (11)$$

IV. COMPARISON OF THE TWO METHODS

The two methods presented above differ in the treatment of an SEP violation. This contrast is illustrated by a comparison of equation (11) with equation (1), after modification of the latter by substitution of equation (2). In atomic units, Method I conserves angular momentum per unit mass J, whereas Method II does not because of the last term in equation (11), a viscosity-like term which is not present in the modified equation (1). Hence, in atomic units, for a planet in a circular orbit around the Sun (at a distance R and with a period P), Method I yields, using Kepler's laws $(2\pi)^2R^3/P^2 = G(t)M_0$ and $J = 2\pi R^2/P$ = constant,

$$\frac{\dot{R}}{R} = \frac{1}{2}\frac{\dot{P}}{P} = -\frac{\dot{G}}{G} , \quad (12)$$

i.e., a decreasing $G(t)$ implies that planetary orbits expand and periods get longer. By contrast, in Method II (in atomic units), the ϕ component of the "in-plane" polar-coordinate form of equation (11) yields $\dot{\phi}r^2\beta_a$ = constant, which, for a circular orbit of radius R and period P, gives

$$2\pi\beta_a R^2/P = \text{constant} , \quad (13)$$

expressing the modified angular momentum conservation law. The corresponding r component of equation (11) gives for a circular orbit

$$\frac{(2\pi)^2R^3}{P^2} = \frac{G_0 M_0}{\beta_a} . \quad (14)$$

From equations (13) and (14), one finds

$$\frac{\dot{R}}{R} = \frac{\dot{P}}{P} = -\frac{\dot{\beta}_a}{\beta_a} = \frac{1}{2}\frac{\dot{G}}{G} , \quad (15)$$

where in the last step we have used the fact that in Method II, the gravitational constant G satisfies $G\beta_a{}^2 = G_0$ = constant (Canuto and Goldman 1982*a*). Therefore, in this method, a decreasing $G(t)$ implies that planetary orbits shrink and periods get shorter.

If there were data sensitive to both the linear and the quadratic terms in $\delta\tau$, and if one of equations (6) and (11) were correct, it would be possible to deduce the correct one. For the data analyzed so far (see, for example, Reasenberg and Shapiro 1976), a useful bound could be placed only on the quadratic term; hence, in this case the two methods are equivalent:

$$(\delta\tau)_{\rm I} \approx \left(\frac{\dot{G}_0}{G_0}\right)\dot{\tau}_0 t^2 , \quad (\delta\tau)_{\rm II} \approx \frac{1}{2}\left(\frac{\dot{\beta}_a}{\beta_a}\right)_0 \dot{\tau}_0 t^2 , \qquad (16)$$

since these two expressions have identical *signatures* and are therefore *equally useful in a search for a non-null coefficient of the quadratic term*, the hallmark of an SEP violation.

This conclusion is in accord with the previous claim by Shapiro and Reasenberg (1980), based on a similar but less general analysis: the goal of "smoking out" the presence of any SEP violation can be achieved equally well using Method I or Method II, if the data are usefully sensitive only to the quadratic term.

The *Viking* data in combination with planetary ranging data may, however, allow the determination of useful values for the coefficients of the linear *and* the quadratic terms, if the above-mentioned problems with systematic errors can be overcome. At present two groups, one at the Harvard-Smithsonian Center for Astrophysics and the other at the Jet Propulsion Laboratory (the latter in collaboration with Canuto and Goldman) are analyzing these data using both methods. More useful results than hitherto obtained should be soon forthcoming at least for the coefficient of the quadratic term in equation (6) or (10).

This work was supported in part by grants to one of us (I. I. S.) from the National Science Foundation, PHY 81-06036, and the National Aeronautics and Space Administration, NSG-7556.

REFERENCES

Adams, P. J., Canuto, V. M., Goldman, I., and Hellings, R. W. 1983, *Phys. Rev. D*, in press.
Canuto, V. M., and Goldman, I. 1982*a*, *Nature*, **296**, 709.
———. 1982*b*, in *IAU Symposium 104, Early Evolution of the Universe and Its Present Structure*, ed. G. O. Abell and G. Chincarini (Dordrecht: Reidel), in press.
———. 1983, *Nature*, **304**, 311.
Dirac, P. A. M. 1938, *Proc. Roy. Soc. Series A*, **165**, 199.
———. 1974, *Proc. Roy. Soc. London*, **A338**, 439.
———. 1979, *Proc. Roy. Soc. London*, **A365**, 19.
Hellings, R. W. 1982, private communication.
Reasenberg, R. D., and Shapiro, I. I. 1976, in *Atomic Masses and Fundamental Constants* 5, ed. J. H. Sanders and A. H. Wapstra (New York and London: Plenum), p. 643.
Shapiro, I. I. 1964, *Phys. Rev. Letters*, **13**, 789.
Shapiro, I. I., and Reasenberg, R. D. 1980, private communication.
Shapiro, I. I., Smith, W. B., Ash, M. E., Ingalls, R. P., and Pettengill, G. H. 1971, *Phys. Rev. Letters*, **26**, 27.
Thorne, K. S., Lee, D. L., and Lightman, A. P. 1973, *Phys. Rev.*, **D7**, 3563.
Will, C. M. 1979, in *General Relativity*, ed. S. W. Hawking and W. Israel (New York: Cambridge University Press), p. 24.

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